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ABSTRACT. This study demonstrates that at large distances from a body moving at a supersonic velocity, there exist two shock waves which follow each other, rather than one wave, as has been supposed. The shapes of these waves and the law of decrease of their intensity are determined. The propagation of spherical shock waves (explosion waves) at large distances from the point of explosion also is investigated.

Shock waves are weak at large distances from their source. Consequently, they have the character of sound waves. However, an ordinary linear approximation is not sufficient for our purposes; it is necessary to examine the properties of low-amplitude sound waves in the second approximation. Below, we shall be concerned with cylindrical waves; however, bearing in mind that at large distances it is possible to consider a cylindrical or spherical wave in each small section as a wave, we shall first mention some properties of waves. /286*

As we know, a travelling wave with an arbitrary amplitude is described by the so-called Riemann solution to the motion equations:

$$x = t[v + c(v)] + f(v),$$

where $f(v)$ is an arbitrary function of gas velocity v, while c is the local velocity of sound which is related to v by means of

$$v = \int \frac{dp}{\rho c} = \int \sqrt{-\frac{\partial V}{\partial p}}\, dp$$

where ρ is the density and V is the specific volume of the gas.

These two formulas implicitly determine the velocity v and the remaining quantities for the wave as a function of x and t, *i.e.*, the wave profile at each given moment of time. When $t = 0$, we have $x = f(v)$, *i.e.*, the inverse function of $f(v)$ determines the wave profile at the initial moment of time.

The quantity

$$u = v + c(v) \tag{1}$$

is the velocity at which the points of the wave profile move. This velocity is variable for different points of the profile. Consequently, the profile will

*Numbers in the margin indicate pagination in the foreign text.

not remain constant, and it will change its shape with time. Let us say that having expressed u as a function of pressure p in the wave, for the derivative we have

$$\frac{du}{dp} = \frac{dc}{dp} + \frac{1}{\rho c}$$

Since

$$c = \sqrt{\frac{\partial p}{\partial \rho}} = V\sqrt{-\frac{\partial p}{\partial V}}$$

a computation results in /287

$$\frac{du}{dp} = \frac{1}{2}\rho^3 c^3 \left(\frac{\partial^2 V}{\partial p^2}\right)_S$$

The adiabatic derivative is $(\partial^2 V/\partial p^2)_S$, where S is the entropy, which is positive for all gases, so that $du/dp > 0$. Thus, the velocity of a given point of the wave profile increases in proportion to the pressure increase at that point. Therefore, the points of compression move forward, while the points of expansion seem to stand still.*

For a low-amplitude wave, the velocity u of the points of the profile in the first approximation will be obtained if we place the velocity $v = 0$ in (1), *i.e.*, $u = c_0$. (Letters with the subscript zero will designate equilibrium values of the quantities.) This corresponds to wave profile movement without a change in shape.

In the next approximation, we have

$$u = c_0 + \frac{\partial u}{\partial p_0} p'$$

or

$$u = c_0\left(1 + \alpha\frac{p'}{p_0}\right), \qquad \alpha = \frac{p_0}{2}\frac{c_0^4}{V_0^3}\left(\frac{\partial^2 V}{\partial p^2}\right)_S \tag{2}$$

where p' is the variable part of the pressure in the wave. For an ideal gas

$$\alpha = \frac{\gamma + 1}{2\gamma} \qquad \text{(for air } \alpha = 0.86\text{)}$$

where $\gamma = c_p/c_v$ is the heat capacity ratio at constant pressure and volume.

*For a more detailed discussion concerning the Riemann solution, see *e.g.*, [1], §77.

When the wave profile is deformed to the point that ambiguity appears in it, we know that a shock wave emerges. Generally speaking, the Riemann solution becomes inapplicable after the formation of discontinuities. However, it is significant that this solution applies for low-amplitude waves in the second approximation examined. It also applies when discontinuities are present. It is possible to be certain of this in the following manner. Velocity, compression and specific volume shocks in a discontinuity are interrelated by the relationship

$$v_2 - v_1 = \sqrt{(p_2 - p_1)(V_1 - V_2)}$$

The change of velocity v along a certain length interval of the x axis in the Riemann solution is equal to the integral

$$v_2 - v_1 = \int_{p_1}^{p_2} \sqrt{-\frac{\partial V}{\partial p}}\, dp$$

A simple computation using series expansion indicates that both of the written expressions only differ from each other in the terms of the third order of smallness. (During computation, it is necessary to bear in mind that the entropy change in the discontinuity is a third-order quantity, while entropy is generally constant in the Riemann solution.) /288

Hence, it follows that the motion in a traveling wave when a discontinuity is present can be described with an accuracy up to the terms of the second order on each side of the discontinuity by the Riemann solution. The appropriate boundary condition will thereby be achieved in the discontinuity itself. In the following approximations, the quantity related to the appearance of the waves reflected from the discontinuity surface will not be indicated.

The place of the discontinuity formation in the wave is determined by the simple geometric condition which can be derived easily using formula (2), and the flow continuity condition of matter in the discontinuity (see [1], §78). The discontinuity is situated in such a way that the area enclosed within the curve depicting the wave profile remains the same as for the ambiguous curve which is determined by the Riemann solution.

Let us now examine a body which is moving steadily at a supersonic velocity U. We shall select the coordinate axis x in the direction of the body's motion, and shall let r be the distance from that axis. At large distances from the body, the velocity potential $\varphi\ (r, z)$ of the gas is determined in the first approximation by the wave equation

$$\frac{1}{c_0^2}\frac{\partial^2\varphi}{\partial t^2} = \frac{\partial^2\varphi}{\partial x^2} + \frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial\varphi}{\partial r}\right)$$

The steady motion condition of the body is

$$\int_{(cb'a')} \chi\, d\tau = \int_{(cb'a')} \chi f'(\chi)\, d\chi$$

Combining both of these equations, we obtain

$$\left(\frac{U^2}{c_0^2}-1\right)\frac{\partial^2\varphi}{\partial x^2}=\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial\varphi}{\partial r}\right)$$

If in the place of x we introduce the variable

$$\tau=\frac{x}{\sqrt{U^2-c_0^2}} \tag{3}$$

we obtain the equation

$$\frac{1}{c_0^2}\frac{\partial^2\varphi}{\partial\tau^2}=\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial\varphi}{\partial r}\right)$$

i.e., the equation of a cylindrical wave, in which τ is the time.

At sufficiently large distances, it is possible to consider a cylindrical wave in each small section as being two-dimensional. Then, the velocity of each point of the wave profile will be determined by formula (2). However, if we wish to use this formula to trace the point shift of the wave profile over long intervals, it is necessary to consider that already in the first approximation the amplitude of the cylindrical wave decreases with distance as $1/\sqrt{r}$.

Introducing the designation

$$\frac{p'}{p_0}=\frac{\chi}{\sqrt{r}} \tag{4}$$

and inserting it into formula (2), we obtain /289

$$\frac{\partial\varphi}{\partial t}+U\frac{\partial\varphi}{\partial x}=0 \tag{5}$$

The first term corresponds to wave motion without a change in profile (digressing from the overall amplitude decrease as $1/\sqrt{r}$), while the second term leads to profile distortion. The quantity δr of this additional displacement of the profile point at a distance from a certain given large r_0 to r is obtained by multiplying by $c_0^{-1}dr$ and integrating from r to r_0 at constant χ. We then obtain

$$u=c_0\left(1+\frac{\alpha\chi}{\sqrt{r}}\right)$$

If we examine the wave profile as the change of p' with τ when r is given, the profile distortion δr will be

$$\delta\tau = \frac{\delta r}{c_0} = \frac{2\alpha\chi}{c_0}\left(\sqrt{r} - \sqrt{r_0}\right) \tag{6}$$

As we know, a diverging cylindrical wave can be written in linear approximation as

$$\varphi = \int_r^\infty \frac{1}{\sqrt{\xi^2 - r^2}} f\left(\tau + \frac{\xi}{c}\right) d\xi \tag{7}$$

The sign of f is the inverse of the usual one which, correspondingly, is that in the given case the wave is propagated from the positive values of τ to the negative ones. Here and below, we shall omit the zero subscript to the values of the quantities for the equilibrium state for brevity.

In our case, the time τ is actually the coordinate x. We shall select the reference origin within the body (at the given moment of time). The regions in front of the body will thereby correspond to positive values of x. Insofar as disturbances are not propagated in the space in front of the body during supersonic motion, it is possible in any case to confirm that $\varphi \to 0$ when $\tau \to \infty$. Furthermore, at sufficiently large distances behind the body, where the disturbances caused by it are small (even on the axis itself $r = 0$) the potential of the diverging wave, which is determined by formula (7), must remain finite when $r = 0$. For convergence of the integral

$$\varphi(0, \tau) = \int_0^\infty f\left(\tau + \frac{\xi}{c}\right) \frac{d\xi}{\xi}$$

it is necessary that $f(\tau) \to 0$ for high negative τ at the lower limit (for high negative τ). Hence, it is easy to conclude that $\varphi \to 0$ also when $\tau \to -\infty$. Conversely, the variable part of pressure in the linear approximation is related to φ by means of the equality $p' = \rho c^2 \partial\varphi/\partial\tau$. Integrating with respect to τ, we consequently obtain

$$\int_{-\infty}^{+\infty} p' d\tau = 0 \tag{8}$$

This means that if there is bunching in the gas (the region $p' > 0$), a rarefaction region must necessarily exist also where $p' < 0$. In this relation, a cylindrical wave (the same applies to a spherical wave) differs significantly from a two-dimensional wave which may consist of only single bunchings or single rarefactions. /290

As we know, a shock wave occurs in a gas when a body moves at supersonic velocity. The gas is motionless in the space in front of this wave, and there is a bunching region directly behind the wave. It follows from what has been stated above that bunching must necessarily be replaced by rarefaction. Consequently, a point must exist at which rarefaction is maximum. Owing to the effect of gradual profile distortion, this point will lag behind those situated behind it. Finally, as a result, ambiguity is obtained, and another shock wave appears.

Thus, we arrive at this result: at least at sufficiently large distances from a moving body, there is not one shock wave (as has usually been supposed), but two shock waves which follow each other. In the first wave, the pressure experiences an upward shock. Then a region of gradual pressure decrease follows and bunching is replaced by rarefaction. After this, the pressure again increases abruptly in the second shock wave.

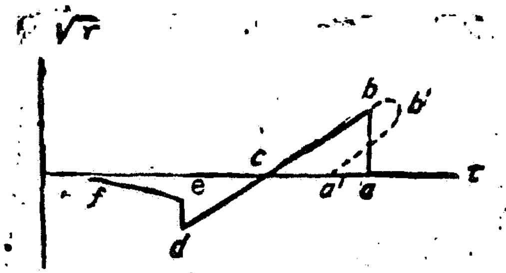

Figure 1.

Figure 1 schematically illustrates (solid line) the resulting picture of pressure p' as a function of τ, *i.e.*, as a function of coordinate x, at a given large value of r. The segment ab corresponds to the first shock wave and de corresponds to the second. In the latter, the pressure only increases up to a certain negative value, while p' becomes equal to zero asymptotically when $\tau \to = \infty$.

Proceeding to a quantitative calculation of the profile illustrated in Figure 1, let us examine the region between both shock waves. Let the function $\tau = f(\chi)$ determine the profile at a certain distance r_0.

Taking into account the effect of profile distortion, we obtain a profile at the distance $r > r_0$ by adding the displacement $\delta\tau$ to τ according to (6).

$$\tau = f(\chi) + \frac{2\alpha}{c}(\sqrt{r} - \sqrt{r_0})\chi . \tag{9}$$

At large r values, the quantity χ is small; and it is possible to write the value of the function $f(\chi)$ in (9) with sufficient accuracy when $\chi = 0$. It is also possible to disregard $\sqrt{r_0}$ in comparison with $\sqrt{r}$. Thus,

$$\tau = \frac{2\alpha}{c}\sqrt{r}\chi + \text{const} \tag{10}$$

The value of coordinate x at point c (Figure 1), where $\chi = 0$, is designated by x_0. Certainly, this value depends on r, according to the law $x_0 = \text{const} - r/c$.

Proceeding to the variables p' and x instead of χ and τ, we have /291

(11)

Thus, the profile segment *bd* proves to be rectilinear. The dotted line in Figure 1 illustrates the profile which is obtained directly by applying the Riemann solution in the entire integral. Actually, there is a discontinuity at a certain point *a*. The position of this point is determined by the geometric condition explained above regarding the equality of areas *a'b'c* and *abc*. Noting that $\chi = 0$ at points a' and *c*, we shall find the following by using (9) for the area *a'b'c*:

i.e., the value which does not prove to be dependent on *r*. Consequently, the very same thing must take place for the area *abc*.

Considering the quantity χ as a function of τ(10), we find without difficulty that the section length l_1 from point *c* (where $p' = 0$) to the leading shock wave ($p' = p_1'$) is proportional to

$$l_1 \sim r^{1/4} \tag{12}$$

Hence, the law of the compression shock p_1' in the leading shock wave as a function of distance will be

$$p_1' = \frac{\text{const}}{r^{5/4}} \tag{13}$$

As for the second discontinuity *ed* (Figure 1), it is easy to show that the ratio of the pressure remaining behind the discontinuity (pressure at point *e*) to the compression shock p_2' in the discontinuity (segment length *ed*) tends to unity when $r \to \infty$. However, this is a relatively slow process. The pressure behind this discontinuity can be considered to be equal to zero only at very large distances *r*. The p_2' compression shock is equal to p_1' by virtue of (8), the entire profile area must be equal to zero.

Let us further discuss the spherically symmetrical shock wave propagation which occurs during an explosion and is viewed far from the explosion. All reasoning here is exactly the same as the reasoning presented above.

During spherical propagation, wave amplitude drops in the first approximation as $1/r$, whereby r is the distance to the center. Therefore, in place of (5), we have

$$u = c\left(1 + \frac{\alpha\chi}{r}\right) \tag{14}$$

for the velocity u of profile point movement, which, in terms of χ, is designated

$$\chi = \frac{p'}{p_0} r \tag{15}$$

Correspondingly, for the profile point displacement δr on a path from a certain r_0 to r, we find

$$\delta r = \alpha\chi \log \frac{r}{r_0}$$

If we consider the wave profile as the change of p' with time t, the distortion δt is /292

$$\delta t = -\frac{\alpha\chi}{c} \log \frac{r}{r_0} \tag{16}$$

Thus, profile distortion of the spherical wave increases with distance according to logarithmic law, *i.e.*, it is much slower than the profile distortion increase of two-dimensional or cylindrical waves. (In the latter case, profile distortion is correspondingly proportional to the first power or root of the distance). Insofar as usual sound absorption related to viscosity and heat conductivity always takes place during the propagation of a real wave in a gas, in view of the slowness of the distortion increase, a spherical sound wave can be absorbed before profile distortion leads to the formation of discontinuities. In particular, if it is a question of the propagation of an explosion shock wave, the second shock wave which might have followed it (as in the case of a cylindrical wave) cannot originate.

In the case under investigation, in place of (9) we have the equation

$$t = f(\chi) - \frac{\alpha}{c}\chi \log \frac{r}{r_0} \tag{17}$$

By expanding $f(\chi)$ in a series with respect to powers of χ, limiting ourselves to terms of the first order, we obtain

$$t = -\frac{\alpha}{c}\chi \log \frac{r}{a} + \text{const} \tag{18}$$

where a is a certain constant. Hence, we again obtain for p' the linear dependence on t as

$$\frac{p'}{p} = \frac{1}{a}\,\frac{c\,(t_0 - t)}{r\,\log(r/a)} \tag{19}$$

Considering the law of conservation of area, for the spherical case we now obtain

$$l_1 \sim \sqrt{\ln \frac{r}{a}}, \qquad p_1' \sim \frac{1}{r\sqrt{\ln \frac{r}{a}}} \tag{20}$$

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